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During the second year of performance we have completed the simulation of the near-Earth asteroids (NEAs) synthetic population to test the optimized survey search strategy. The network of users contributing computing resources has grown continuously, well beyond the level necessary to secure the computational power required by this investigation. One study has been published on the possibility to detect NEAs with the LISA mission. We have also developed the main components of the code that will be used the forthcoming months to test on real data the search strategy with the Catalina Sky Survey (CSS).

We have completed the numerical simulations where a population of synthetic NEAs was searched by one or two competing simulated surveys using different search strategies. The simulations covered periods between 5 and 20 years, and were as realistic as possible, including variable detection efficiency as a function of apparent magnitude and apparent velocity, efficiency drop close to full moon, and earth rotation effects. The results show that it is possible to improve over standard search strategies, especially on the short term, i.e. when the improved search strategy is used for a few months. On the longer term, the advantage of the improved search strategy is slowly reabsorbed, partly because the decreasing number of asteroids yet to be discovered. The emerging patter seems to suggest that surveys should balance between periods adopting regular search strategies (opposition, low solar elongation) with periods adopting optimized search, that points far from the ecliptic at favorable moments.

Most of the software that will be used to test the optimized search strategy using real data was developed this year, and will be deployed in the forthcoming months. It will generate the first optimized search strategy and will be concretely tested at CSS.

All the computational power required by this project have been contributed by volunteers around the world. Our computing network includes now more than 20,000 computers, made available by more than 10,000 volunteers. This has allowed our simulations to reach a level of detail and accuracy that would not have been otherwise possible.

We have used our code and computational power to determine if the NASA/ESA mission LISA, originally designed to reveal gravitational waves of astrophysical origin, will be able to detect passages of NEAs nearby the LISA spacecraft. We find that an average of two events per year should produce a detectable signal, including all asteroids larger than about

100 meters. It is of particular interest the possibility of higher event rates due to smaller asteroids, that could shed some light on the size-frequency distribution at smaller sizes. Also, passages of known asteroids producing a detectable signal will allow to put strong constraints on the asteroid mass. These results were published in Tricarico (2009), *"Near-earth asteroids detection rate with LISA,"* Classical and Quantum Gravity 26, 085003, and a copy of this paper is attached to this report.

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Near-earth asteroids detection rate with LISA

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Abstract
The LISA space mission, designed to monitor low frequency gravitational waves, is also sensitive to passages of asteroids nearby one of its three spacecrafts. We report the expected rate of detections of asteroid passages, using the known catalog of asteroids and a modeled population. The method adopted consists of determining for each known asteroid the critical encounter distance capable of producing a detectable event, and then computing the rate of encounters within this distance. Results are then scaled to the modeled population using its differential distribution in absolute magnitude, correcting for selection effects. We find that an average of 2.0 ± 0.1 events per year at a signal-to-noise ratio of 1 will be detected by LISA, including all the asteroids in the modeled population with absolute magnitude $H < 22$, roughly equivalent to all asteroids with a diameter larger than 100 m.

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1. Introduction

The LISA mission [1] will monitor low frequency gravitational waves using three spacecrafts in an earth-like orbit around the sun, trailing the earth by 20°. The spacecrafts will maintain a triangular configuration with an average side length of five million kilometers, and perform laser interferometric distance measurements.

It has been shown [2] that the passage of one asteroid in the neighborhood of one of the LISA spacecrafts can generate a signal detectable with the standard LISA data analysis, and this has led to the issue of quantifying the rate of such events. In this paper, we report the expected detection rate for the known population of near-earth asteroids (NEAs). Our approach consists of determining, for each known NEA, the rate of close approach within a detectable distance. Results are then scaled to a modeled population using its differential distribution in absolute magnitude, correcting for selection effects. This method is statistic in nature because the final orbit of the spacecrafts is not entirely known, and will remain uncertain until their launch.

2. Method

Our analysis of the rate of detection events of asteroid passages by LISA spacecrafts starts from the results in [2] that we briefly review here. The square modulus of the interferometer relative Doppler frequency shift signal $\tilde{X}_1(f)$ generated by the passage of an asteroid in the neighborhood of one spacecraft is

$$|\tilde{X}_1(f)|^2 = \left(\frac{4GM}{cV^2}\right)^2 \left[K_1^2(\zeta)\sin^2\theta\cos^2\phi + K_0^2(\zeta)\cos^2\theta\right], \tag{1}$$

where f is the frequency, G is the universal gravitational constant, M is the mass of the asteroid, c is the speed of light, V is the relative velocity between asteroid and spacecraft, $K_n(x)$ are the modified Bessel functions of the second kind, $\zeta = 2\pi f D/V$, and D is the impact parameter. The angles (θ, ϕ) are the spherical coordinates of the unit vector $\vec{n}_1 = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ parallel to the line connecting the two other spacecrafts [3] in the reference frame with the origin located on the spacecraft, the z-axis parallel to V, and the relative trajectory of the asteroid contained in the (x, z) plane. The expression for $\rho(f)$, the differential signal-to-noise ratio (SNR), is then

$$\rho^2(f) = 4\frac{\langle|\tilde{X}_1(f)|^2\rangle}{S_{\tilde{X}_1}(f)}, \tag{2}$$

where the average $\langle|\tilde{X}_1(f)|^2\rangle$ is taken over distribution of angles (θ, ϕ) and relative velocities V characterizing the asteroid passage, and $S_{\tilde{X}_1}(f)$ is the one-sided spectral density of the residual noise. Finally, the expression for the SNR is

$$\mathrm{SNR}^2 = \int_{f_c}^{\infty} \rho^2(f)\,\mathrm{d}f, \tag{3}$$

where f_c is a cutoff frequency. This completes our review of [2].

The formalism we use to compute the rate η of passages of an asteroid within a distance D from the spacecraft was developed in [4] and assumes that for each of the two objects $(k = 1, 2)$, three orbital elements are constant (semi-major axis a_k, eccentricity e_k, inclination i_k) while the three remaining angles (longitude of the ascending node, argument of the perihelion, mean anomaly) can assume any value between 0 and 2π radians. This applies to the case studied here because for the LISA spacecrafts only the three orbital elements (a_k, e_k, i_k) have a predetermined value, while the three angles will be determined only when the mission will be operating, as the only constraint is that the longitude of the ascending node and the mean anomaly are shifted by $2\pi/3$ radians in the three spacecrafts in order to maintain a triangular configuration [5]. Thus, in absence of specific values, we choose to use a statistical approach where any value of the angles is equally likely. This forces us to apply the same simplification to NEAs: if we do not know the exact position of a spacecraft, then the exact position of a NEA cannot be used to determine their relative distance D. The close approach rate is then [4]

$$\eta = \int_U S_1 S_2 V \sigma\,\mathrm{d}U = \sigma \int_U S_1 S_2 V\,\mathrm{d}U, \tag{4}$$

where U is the volume accessible to both objects, S_k is the spatial density function of each object and V the relative velocity. The cross section $\sigma = \pi D^2\left(1 + V_e^2/V^2\right) = \pi D^2$ can be moved out of the integral because the escape velocity V_e of the asteroids is much smaller than the encounter velocity V. The expression for S_k is given by [4]:

$$S_k = \frac{1}{2\pi^3 R a_k \sqrt{(\sin^2 i_k - \sin^2 b)(R - q_k)(Q_k - R)}} \tag{5}$$

where R is the heliocentric distance, $\sin b = R_z/R$, R_z is the distance from the ecliptic plane, $q_k = a_k(1 - e_k)$ is the perihelion distance, and $Q_k = a_k(1 + e_k)$ is the aphelion distance. The volume U is defined by $q_U \leqslant R \leqslant Q_U$ and $\sin b \leqslant \sin b_U$, where $q_U = \max\{q_k\}$, $Q_U = \min\{Q_k\}$, $b_U = \min\{b_k\}$ and $b_k = \min\{i_k, \pi - i_k\}$. The spatial density function clearly expresses integrable singularities at the extremes of the domain of R and b, where the orbital motion imposes a change of sign of the time derivative of these variables. For any given value of the pair (R, b) the velocity of one object can assume four distinct values, and consequently the relative velocity V assumes sixteen values. A method to determine all the possible values of V is described in [6].

The use of this approach has two main advantages. First, by performing the integration of equation (4) using a numerical Monte Carlo technique, we can keep track of the distribution of the angles (θ, ϕ) and of the relative velocities V, and this is necessary to evaluate the average $\langle|\tilde{X}_1(f)|^2\rangle$. Second, in equation (4) we have that $\eta \propto D^2$, and this allows us to easily determine the rate for different impact parameters.

One could argue that for the NEAs the information relative to the three angles should be retained as it can affect the distribution of (θ, ϕ) and V. However this variant would require significant changes in the formalism in order to preserve probability normalization, and in most cases would only introduce minor corrections to the distribution of V, while equations (1) and (2) are mainly sensitive to its mean value.

Now, for each NEA we have that the orbital elements (a_k, e_k, i_k) uniquely determine η up to a factor D^2, and the distribution of the triplet (θ, ϕ, V) leaves only D as independent variable in the SNR formula equation (3), in a strictly monotonic relation that can be easily inverted numerically. By fixing a target SNR value, we can derive a value of D satisfying equation (3), and then determine the events rate with equation (4).

3. Results

Known NEAs

We have studied the possibility of detection using the catalog of known NEAs published by the Minor Planet Center (MPC), including 5613 objects as of December 16th, 2008. Of these asteroids, only 3326 (59%) have orbits that can intersect the volume covered by the three LISA spacecrafts. The orbit assumed for the spacecrafts is the following [5, 7]:

$$\begin{aligned} a &= 1\ \text{AU} \\ e &= L/(2\sqrt{3}a) = 0.009\,648 \\ i &= L/(2a) = 0.9575^\circ \end{aligned} \qquad (6)$$

with $L = 5 \times 10^6$ km the nominal distance between two spacecrafts. For each asteroid, we perform the integral in equation (4) using a numerical Monte Carlo technique, that typically requires the evaluation of the integrand at 10^3 points (R, b) to achieve a target relative accuracy of 0.05. While performing the integration, the values of the triplet (θ, ϕ, V) are saved along with their relative probability, and form the set over which we later compute the average of the square modulus of the signal. The above choice of accuracy represents the sweet spot between keeping the overall study computationally feasible and obtaining results that are consistent within the scope of this study.

The values of V have a typical dispersion of the order of 1 km s^{-1} with a distribution that shows two peaks near the edges of the range (see figure 1). The two peaks are typically due to (R, b) being close to the singularities of the spatial density function S_k of the LISA spacecrafts. By summing the close approach probability over all the NEAs, we can have a

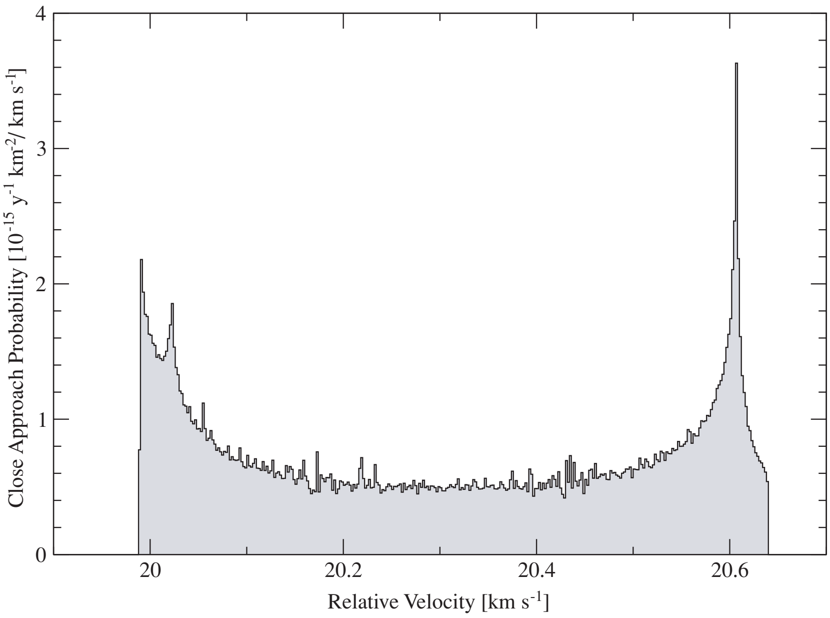

Figure 1. Relative velocity distribution between asteroid 88254 ($a = 1.182$ AU, $e = 0.629$, $i = 1.524^\circ$) and one LISA spacecraft. The integrated close approach rate is $4.5 \times 10^{-16} (\pm 5\%)$ per year per km^2.

snapshot of the distribution in V and θ. In figure 2 we show that the range of relative velocities extends from below 1 to beyond 35 km s^{-1}. In figure 3 the distribution in θ is displayed, with a *volcano* shape peaking between 60° and 120°. The distribution in ϕ is flat.

The conversion from asteroid absolute magnitude H, provided by the MPC catalog, to asteroid mass M, is performed using the relation [8]

$$d = 1329 \text{ km} \times 10^{-H/5} p_V^{-1/2}, \tag{7}$$

where d is the mean diameter of the asteroid in km, and p_V is its albedo. The mass is then obtained assuming a sphere of uniform density ρ. The values used are $p_V = 0.154$ and $\rho = 2.6$ g cm^{-3} as suggested in [9] as weighted average values for the NEAs population.

The impact parameter D is now determined, for each NEA, by numerically inverting equation (3) to obtain a desired SNR value, using root finding algorithms. The expression adopted for $S_{\tilde{X}_1}(f)$ during the calculations is the same adopted in [2], with identical parameters, including the cutoff frequency $f_c = 10^{-5}$ Hz. We choose to work with two SNR values, 1 and 3, to show how our results depend on the sensitivity limits that will be used when analyzing LISA data. The resulting values of D are displayed in figure 4. For km-class asteroids the value of D is of the order of 1 million kilometers, making it possible in principle for one NEA to perturb more than one LISA spacecraft during the same passage. However, a fit of the data shows that the characteristic interaction time $\tau = D/V$ follows the simple relation:

$$\tau = 2.3 - (H - 13)/5, \tag{8}$$

where τ is in days, for $H < 22$ and SNR equal to 1 with a dispersion of ± 0.2 days, and about 0.2 days shorter for SNR equal to 3. This means that for most NEAs with $H < 22$ the value of τ is of the order of 1 day, and in this short time the NEA will not travel far enough to perturb a second spacecraft. So multiple events by the same NEA and during the same passage are possible, but at epochs separated by several times τ, confirming the assumption

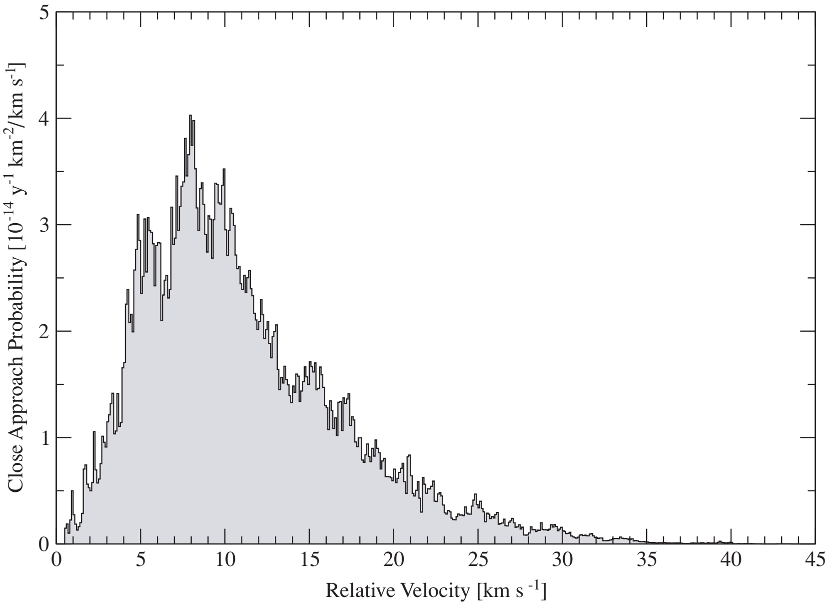

Figure 2. Relative velocity distribution summed over the 3326 known NEAs intersecting the orbit of one LISA spacecraft.

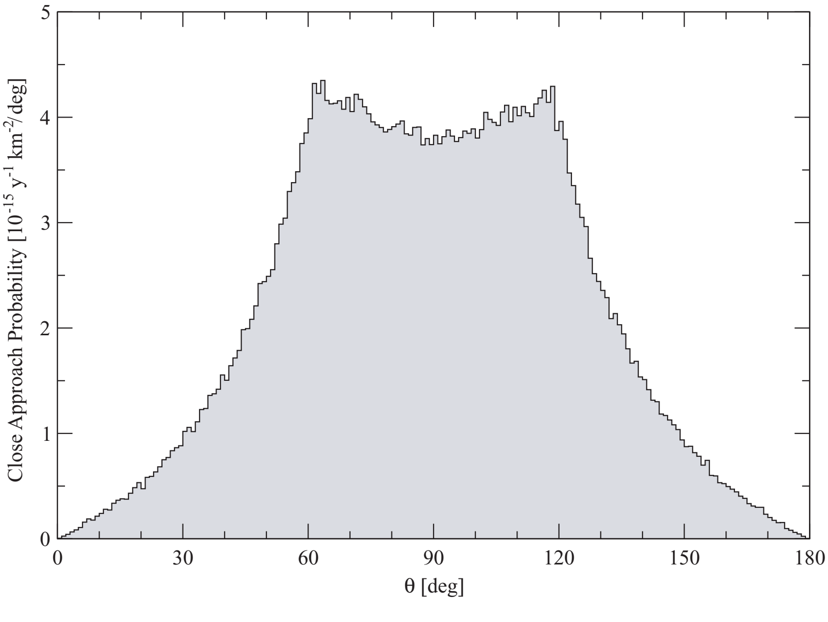

Figure 3. Cumulative distribution of the θ angle summed over the 3326 known NEAs intersecting the orbit of one LISA spacecraft. The *volcano* shape with peaks at 60° and 120° is due to the nominal inclination of 60° between the plane containing the three LISA spacecrafts and the ecliptic plane.

that each event involves only one spacecraft [2]. For simplicity we will ignore the statistical correlation effects of multiple detections during the same passage.

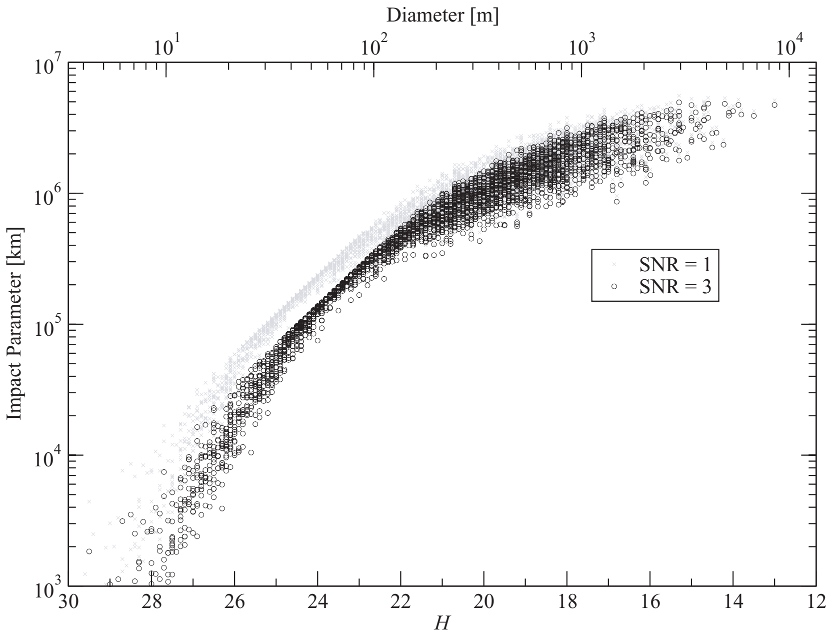

Figure 4. Impact parameter for SNR 1 (gray cross) and 3 (black circle) relative to all known NEAs studied. For a given value of absolute magnitude H and correspondent diameter, a large variability in the impact parameter can be observed, due to the wide range of relative velocities and encounter geometry angles.

We can now estimate how often a given known NEA experiences a close approach capable of generating a detectable signal. We find that summed over all known NEAs, this rate is of 0.32 ± 0.02 events per year per spacecraft for SNR equal to 1, and of 0.23 ± 0.01 events per year per spacecraft for SNR equal to 3, where the uncertainty used here comes from the target relative accuracy of 0.05 used thorough the computations. The contribution to the total rate by NEAs is showed in figure 5, where asteroids are grouped according to their H.

Modeled NEAs population

We can now estimate the detection rate for the modeled NEAs population. In order to do so, we compute to total detection rate due to known NEAs within a short range in H, and then scale it to the number of modeled NEAs within the same range, including a correction coefficient related to selection effects. Correcting for bias in the NEAs population is not a simple matter, as it requires detailed knowledge of surveys performance [10], or detailed numerical integrations of the dynamics of asteroids from their source regions [11, 12]. In this context, the main selection effect is that asteroids with larger H tend to have orbits close to the earth's one, because discovered only when very close to the observer. This tends to over-estimate the close approach rate in the modeled population, because orbits with one of the apsides close to 1 AU and with low inclinations ($i \simeq i_{\mathrm{LISA}} = 0.9575^\circ$) are near the integrable singularities of the spatial density function S_k, see equation (5). To correct for this bias, we choose a subset of known NEAs to provide a reference orbital distribution, and compute the average close approach rate ϱ_{ref} at an unitary distance D for it, see equation (4). This average rate should not change significantly for NEAs within different H ranges, and as

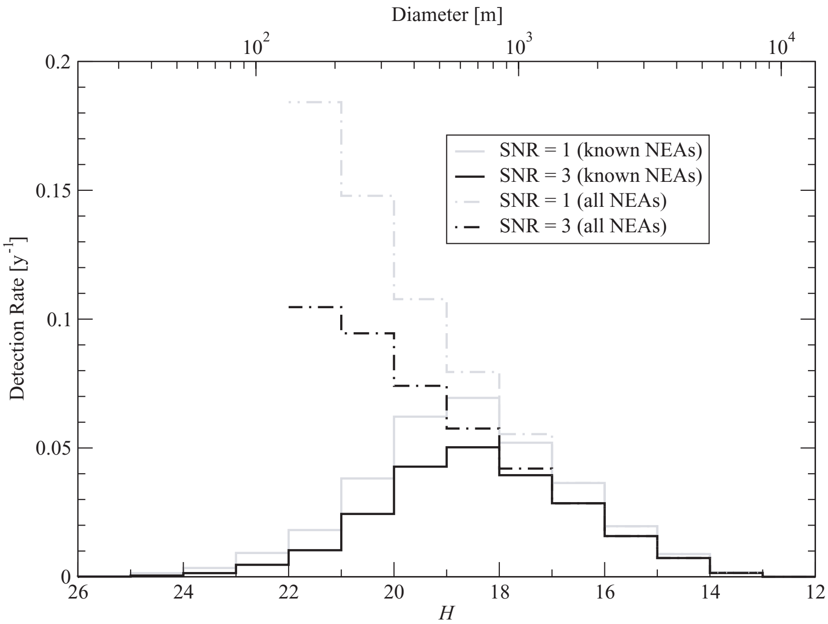

Figure 5. Contribution to the total detection rate by known NEAs (solid line) and modeled NEAs population (dashed-dotted line), at SNR 1 (gray) and 3 (black).

such can be used as a simple bias indicator. If ϱ_j is the average rate relative to NEAs with $j \leqslant H < j+1$, we have that the bias correction factor is $\varepsilon_j = \varrho_{\mathrm{ref}}/\varrho_j$. To choose the reference subset of NEAs, we note that the MPC catalog used here can be considered to be an observationally complete set up to $H < 16$, but this would not provide a large enough statistical sample (78 NEAs), so we choose as reference subset NEAs with $H < 17$ (207 NEAs), close to be observationally complete and with adequate statistical robustness. The correction factor is then: $\varepsilon_{17} = 0.95, \varepsilon_{18} = 0.76, \varepsilon_{19} = 0.59, \varepsilon_{20} = 0.52, \varepsilon_{21} = 0.42$. We adopt the following differential distribution for the modeled NEAs population [10–12]:

$$\mathrm{d}N_{\mathrm{NEA}}(H) = C_{\mathrm{NEA}} \times 10^{\gamma(H-H_0)}\,\mathrm{d}H \tag{9}$$

where $\gamma = 0.35$ for $H < 22$, $C_{\mathrm{NEA}} = 13.26$ and $H_0 = 13$. For $H > 22$ no reliable differential distribution is available in the literature, so our estimates are limited to the NEAs with $H < 22$.

We find that the expected rate for the modeled NEAs population is of 0.65 ± 0.03 events per year per spacecraft for SNR equal to 1, and of 0.44 ± 0.02 events per year per spacecraft for SNR equal to 3, including the modeled NEAs population up to $H = 22$, roughly equivalent to 100 m class NEAs. The contribution to the total detection rate by unobserved 100 m class NEAs is now determinant, as showed in figure 5.

4. Conclusions

We have assessed the rate of NEAs detections with LISA, using both the known NEAs and a modeled NEAs population. We expect an average of 2.0 ± 0.1 events per year at SNR 1, and about 1.3 ± 0.1 events per year at SNR 3. The uncertainty on these figures reflects only the overall accuracy goal adopted during our computations, and does not include possible deviations of the NEAs population from the differential distribution equation (9). About 50% of the detection events are expected to be due to NEAs known today, and this fraction will increase considerably by the time the LISA mission will operate, making it very likely to associate events to known NEAs. With an expected mission duration of 5 to 8 years, asteroid detections will provide very valuable data, allowing indirect measurements of the mass of the asteroid every time an event can be associated with a known NEA.

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